# Exam II from Spring 2017

1. (a) For what $p \geq 1$ is $x \to \left(\sum_i |x_i|^p\right)^{1/p}$ m-strongly convex for some $m > 0$? $\underline{M}$-smooth for some $\underline{M} < +\infty$.

Neither

$|x|$

(b) $x \to \|x\|_1^2 = \left(\sum_{i=1}^{d} |x_i|\right)^2$

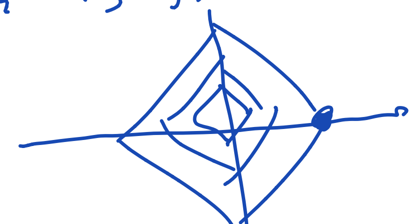
$(t + 1 - t)^2 = 1^2$

$(1, t, 0, \dots) \to (1 + |t|)^2$

Neither

$(x, 0, \dots, 0) \to x^2$

$(t, 1-t, 0, \dots, 0) \quad 0 \leq t \leq 1$



2. $(f_t : t \geq 0)$ random:

$f_0$ fixed

$f_{t+1} = (0.9) f_t + W_t$ independent

$E[W_t] = 0, \quad E[W_t^2] \leq 1$ all $t$

$f_{t+1} = f_t - \alpha_t g(f_t, \xi_t)$

Assumption 1

(i) $\langle \nabla \Gamma(f_t), E_{\xi_t} g(f_t, \xi_t) \rangle \geq \mu \|\nabla \Gamma(f_t)\|^2$

(ii) $E[\|g(f_t, \xi_t)\|^2] \leq B + B_G \|\nabla \Gamma(f)\|^2$

$E(\Gamma(f_t)) - \Gamma^* \leq \frac{\alpha M B}{2 m \mu} + (1 - \alpha m \mu)^{t-1} (\Gamma(f_1) - \Gamma^* - \dots)$

$\to \frac{M B}{2 m \mu}$

Find upper bound on $\frac{1}{2} E[f_t^2] \leq \frac{(0.1)(100)}{2} + 1$

$E[f_t^2] \leq 10 + (0.9)^t (f_0^2 - 9)$

$f_{t+1} = f_t - (0.1)\underbrace{(f_t + 10 W_t)}_{g(f_t, \xi_t)} \quad \alpha_t = 0.1$

$E[g(f_t, \xi_t)] = f_t$

$M = 1$

$m = 1$

Let $\Gamma(f) = \frac{1}{2} f^2 \quad \Gamma' = f$

$\underline{\mu = 1}$ $\quad \langle \nabla \Gamma, Eg \rangle = \langle f, f \rangle = f^2 \geq 1 \cdot |f|^2$

$E_f[g^2] = E[(f + 10 W_t)^2] \leq f^2 + (100) \quad B = 100, \; B_G = 1$

(b) Suppose instead $|W_t| \le 1$ all $t$, arbitrary

Bound $E[f_t^2]$ in terms of $f_0, t$

$$|f_1| = |(0.9) f_0 + \underbrace{W_0}_{|W_0| \le 1}| \le (0.9)|f_0| + 1$$

$$|f_{t+1}| \le (0.9)|f_t| + 1$$

$$|f_t| \le b_t = (0.9)^t |f_0| + \sum_{s=1}^{t} (0.9)^{t-s} = 10 + (0.9)^t(|f_0| - 10)$$

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$(Z, P, \mathcal{F}, \ell)$

$\mathcal{F}$ - closed convex subset $\mathcal{H}$

$\ell$ m-s.c. $\ell(f,z)$
L-Lipschitz

$$\hat{f}_n = \arg\min_f \frac{1}{n}\sum_{i=1}^{n} \ell(f, z_i)$$

Then $L(\hat{f}_n) - L^* \le \dfrac{2L^2}{\delta m n}$

w/p $\ge 1-\delta$

(c) $\mathcal{F} = [-1, 1]$

$Z = [-1, 1] \leftarrow$

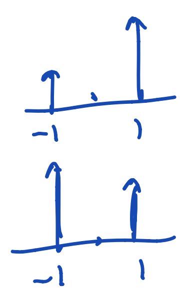
$\ell(f, z) = (f - z)^2$

$Z^n = (Z_1, \dots, Z_n)$

$\hat{f}_n = A(Z^n)$

$$P_1 = \frac{1+h}{2}\delta_1 + \frac{1-h}{2}\delta_{-1}$$

$$P_{-1} = \frac{1-h}{2}\delta_1 + \frac{1+h}{2}\delta_{-1}$$




$P_1^n$ Show the generalization loss for $P_1$ is

$$L_1(f) = 1 - h^2 + (h - f)^2$$

$$L_1(f) = \left(\frac{1+h}{2}\right)\underbrace{(f-1)^2}_{z=1} + \left(\frac{1-h}{2}\right)\underbrace{(f+1)^2}_{z=-1}$$

See 2017 Exam 1 sol for remainder